**Showing the disconnectedness design from the projective matrices of the DF**

The orthogonal projector of Between Runs stratum can be written as

where denotes the projection matrix of runs and can be computed from . The matrix is the n-by-n averaging matrix where all elements contain 1/n.

The orthogonal projector of Within Runs stratum can be written as

where I is the n-by-n identity matrix.

The degrees of freedom (DF) associated with each of these two stratum can be computed from their trace, i.e. the DF associated with Between Runs stratum is

where denotes the total number of runs. The DF associated with Within Runs stratum is

The orthogonal projector of the Between Animals Between Runs stratum can be written as

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The DF of the Between Animals Between Runs stratum can then be computed as

The orthogonal projector of the Between Animals Within Runs stratum can be written as

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The DF of the Between Animals Between Runs stratum can then be computed as

where can be expanded as

where denotes the animal incidence matrix with respect to runs, is animal concurrence matrix with respect to runs, denotes number of technical replicates and denotes the block size.

**If the allocation of animals to runs is binary**, i.e. every element in is one or zero, then should equal and

Hence, the DF of the Between Animals Between Runs stratum and Within Runs are

and

respectively.

For the cases of , DF of the Between Animals Between Runs stratum and Within Runs are 0 and respectively. This means the allocation of animals to runs is connected where all the information associated with animals are in the Within Runs stratum. Note that, this case is an example of completely randomised block design.

In addition, for the cases of , and the allocation of animals to runs is still binary. The DF of the Between Animals Between Runs stratum and Within Runs are then becomes 1 and . This means the allocation of animals to runs is now disconnected, where one of DF associated with Animals is in the Between Runs stratum. This also shows that even if the allocation is binary, it can also disconnected if is devisable by .

**If the allocation of animals to runs is non-binary**, i.e. some elements in is larger than one; and should be larger than .

For example, if the design is with and the allocation of animals to run is binary. The design of animal allocation to runs and tags can be express in a table as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | A | B | C | D |
| 2 | B | C | D | A |
| 3 | C | D | A | B |
| 4 | D | A | B | C |

where the upper case letter denotes animal ID. The animal incidence matrix with respect to runs is

and the animal occurrence matrix with respect to runs is then

the trace of this matrix is 16.

**If the allocation of animals to runs is non-binary**, then the design of animal allocation to runs and tags can be express in a table as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | A | B | A | B |
| 2 | B | A | B | A |
| 3 | C | D | C | D |
| 4 | D | C | D | C |

The animal incidence matrix with respect to runs is

and the animal occurrence matrix with respect to runs is then

the trace of this matrix is 32, which is larger than the 16 of the previous binary design.

**If is larger and not devisable by** , there can be two cases on whether the allocation of animals to runs is binary or non-binary.

For example, if the design is with and . The allocation of animals to run is binary. The design of animal allocation to runs and tags can be express in a table as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | A | B | C | D |
| 2 | E | F | A | B |
| 3 | C | D | E | F |

The animal incidence matrix with respect to runs is

[,1] [,2] [,3]

[1,] 1 1 0

[2,] 1 1 0

[3,] 1 0 1

[4,] 1 0 1

[5,] 0 1 1

[6,] 0 1 1

and the animal occurrence matrix with respect to runs is then

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 2 2 1 1 1 1

[2,] 2 2 1 1 1 1

[3,] 1 1 2 2 1 1

[4,] 1 1 2 2 1 1

[5,] 1 1 1 1 2 2

[6,] 1 1 1 1 2 2

the trace of this matrix is 12 which is not divisible by = 8. The next divisible number by 8 is 16.

**If the allocation of animals to runs is non-binary**, and example of a design of animal allocation to runs and tags can be express in a table as follows

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | A | B | C | D |
| 2 | B | A | D | C |
| 3 | E | F | E | F |

Run 1 and 2 contain Animal A, B, C and D and Run 3 contains Animal E and F. The animal incidence matrix with respect to runs is

[,1] [,2] [,3]

[1,] 1 1 0

[2,] 1 1 0

[3,] 1 1 0

[4,] 1 1 0

[5,] 0 0 2

[6,] 0 0 2

and the animal occurrence matrix with respect to runs is then

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 2 2 2 2 0 0

[2,] 2 2 2 2 0 0

[3,] 2 2 2 2 0 0

[4,] 2 2 2 2 0 0

[5,] 0 0 0 0 4 4

[6,] 0 0 0 0 4 4

The trace of this matrix is 16 and it is divisible by 8. Using the equations described earlier. DF associated with the Between Animal Between and Within Runs strata are

and

The design can be disconnected even more giving

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Runs | Tag | | | |
| 114 | 115 | 116 | 117 |
| 1 | A | B | A | B |
| 2 | C | D | C | D |
| 3 | E | F | E | F |

Run 1 only contains Animal A and B, Run 2 only contains Animal C and D and Run 3 contains Animal E and F.

The animal incidence matrix with respect to run can be shown as follows

[,1] [,2] [,3]

[1,] 2 0 0

[2,] 2 0 0

[3,] 0 2 0

[4,] 0 2 0

[5,] 0 0 2

[6,] 0 0 2

and the animal occurrence matrix with respect to runs is then

[,1] [,2] [,3] [,4] [,5] [,6]

[1,] 4 4 0 0 0 0

[2,] 4 4 0 0 0 0

[3,] 0 0 4 4 0 0

[4,] 0 0 4 4 0 0

[5,] 0 0 0 0 4 4

[6,] 0 0 0 0 4 4

The trace of this matrix is 24 and it is divisible by 8. Using the equations described earlier. DF associated with the Between Animal Between and Within Runs strata are

and

In summary, the formula for computing the DF associated with the Between Animal Between and Within Runs strata are

and

where .

If the number of animals is divisible by the tag number, then the DF associated with the Between Animals Between Runs and Within Runs can be derived directly from

and

Otherwise, if the number of animals is not divisible by the tag number, then we can use the ceiling of number of animals divided by the tag number, i.e.

and

It is also important to examine the level of connectedness on the allocation of animals to runs. This has shown that can affect the structure of the occurrence matrix and its trace, which is used to determine the DF associated with the Between Animals Between Runs and Within Runs.